Velammal College of Engineering and Technology, Madurai – 625009

Department of Information Technology

Unit I – Amplitude Modulation

Introduction to Communication Systems - Modulation – Need for Modulation - Types – AM, DSBSC & VSB – Generation and Demodulation, Frequency and Phase Modulation – Modulation and Demodulation - Comparison of Analog Communication Systems (AM – FM – PM)

CO1: Apply analog communication techniques.

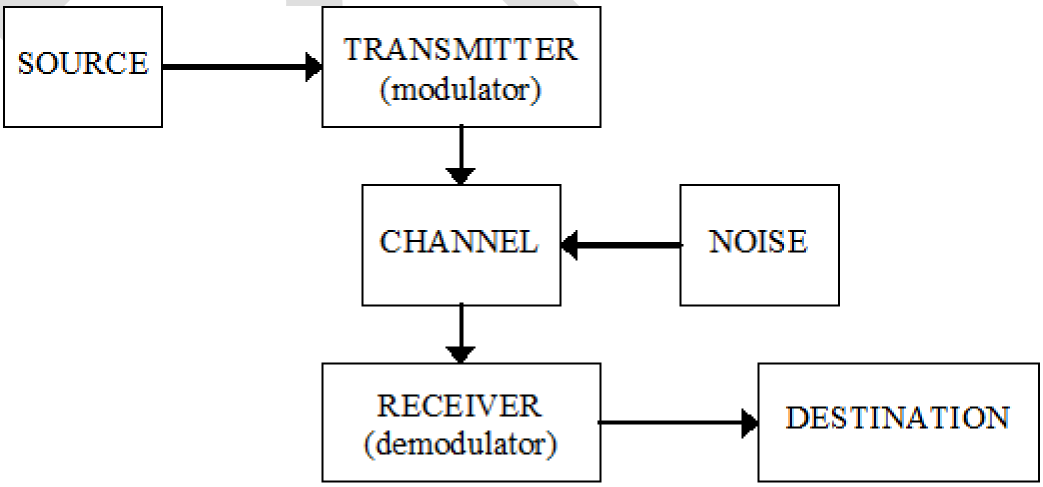
AMPLITUDE MODULATION

Introduction to Communication System

Communication is the process by which information is exchanged between individuals through a medium.

Communication can also be defined as the transfer of information from one point in space and time to another point.

The basic block diagram of a communication system is as follows.



- **Transmitter:** Couples the message into the channel using high frequency signals.
- **Channel:** The medium used for transmission of signals
- **Modulation:** It is the process of shifting the frequency spectrum of a signal to a frequency range in which more efficient transmission can be achieved.
- **Receiver:** Restores the signal to its original form.

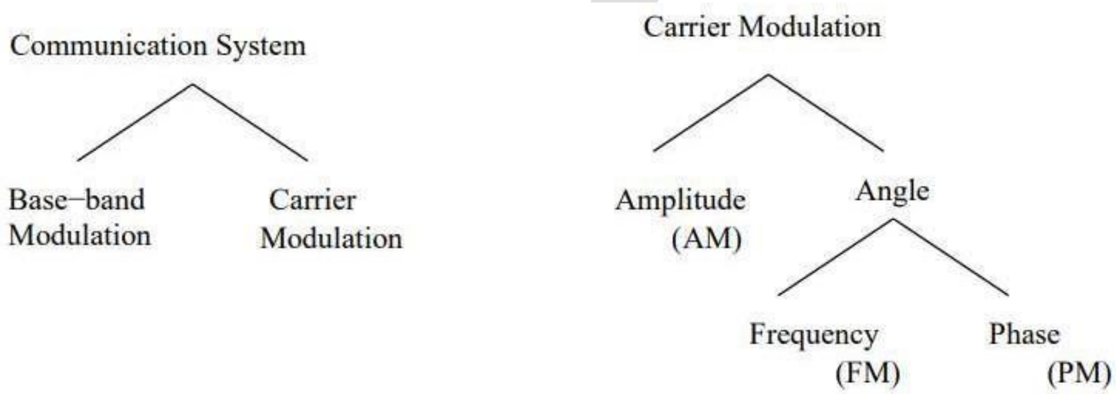
- **Demodulation:** It is the process of shifting the frequency spectrum back to the original baseband frequency range and reconstructing the original form.

Modulation:

Modulation is a process that causes a shift in the range of frequencies in a signal.

• Signals that occupy the same range of frequencies can be separated.

• Modulation helps in noise immunity, attenuation - depends on the physical medium. The below figure shows the different kinds of analog modulation schemes that are available



Modulation is operation performed at the transmitter to achieve efficient and reliable information transmission.

For analog modulation, it is frequency translation method caused by changing the appropriate quantity in a carrier signal.

It involves two waveforms:

- A modulating signal/baseband signal – represents the message.
- A carrier signal – depends on type of modulation.

• Once this information is received, the low frequency information must be removed from the high frequency carrier. This process is known as “Demodulation”.

Need for Modulation:

- Baseband signals are incompatible for direct transmission over the medium so, modulation is used to convey (baseband) signals from one place to another.
- Allows frequency translation:
    - Frequency Multiplexing
    - Reduce the antenna height
    - Avoids mixing of signals
    - Narrow banding
- Efficient transmission

- Reduced noise and interference

Types of Modulation:

Three main types of modulations:

Analog Modulation

- **Amplitude modulation**
  Example: Double sideband with carrier (DSB-WC), Double- sideband suppressed carrier (DSB-SC), Single sideband suppressed carrier (SSB-SC), vestigial sideband (VSB)
- **Angle modulation (frequency modulation & phase modulation)**
  Example: Narrow band frequency modulation (NBFM), Wideband frequency modulation (WBFM), Narrowband phase modulation (NBPM), Wideband phase modulation (NBPM)

Pulse Modulation

- Carrier is a train of pulses
- Example: Pulse Amplitude Modulation (PAM), Pulse width modulation (PWM) , Pulse Position Modulation (PPM)

Digital Modulation

- Modulating signal is analog
  - Example: Pulse Code Modulation (PCM), Delta Modulation (DM), Adaptive Delta Modulation (ADM), Differential Pulse Code Modulation (DPCM), Adaptive Differential Pulse Code Modulation (ADPCM) etc.
- Modulating signal is digital (binary modulation)
  - Example: Amplitude shift keying (ASK), frequency Shift Keying (FSK), Phase Shift Keying (PSK) etc

Amplitude Modulation (AM)

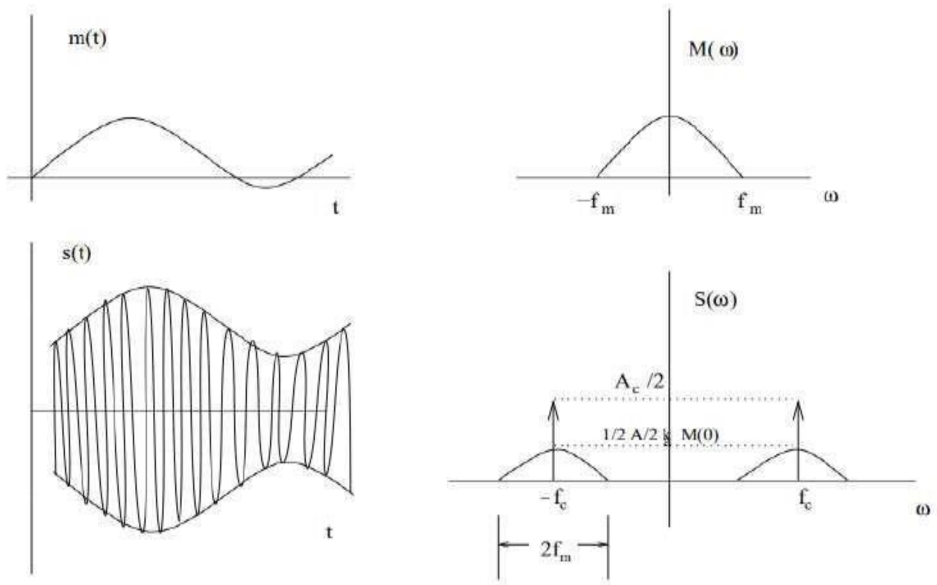
Amplitude Modulation is the process of changing the amplitude of a relatively high frequency carrier signal in accordance with the amplitude of the modulating signal (Information).

The carrier amplitude varied linearly by the modulating signal which usually consists of a range of audio frequencies. The frequency of the carrier is not affected.

- Application of AM - Radio broadcasting, TV pictures (video), facsimile transmission
- Frequency range for AM - 535 kHz – 1600 kHz
- Bandwidth - 10 kHz

Various forms of Amplitude Modulation

- Conventional Amplitude Modulation (Alternatively known as Full AM or Double Sideband Large carrier modulation (DSBLC) /Double Sideband Full



Carrier (DSBFC)

• Double Sideband Suppressed carrier (DSBSC) modulation

• Single Sideband (SSB) modulation

• Vestigial Sideband (VSB) modulation

Time Domain and Frequency Domain Description

It is the process where, the amplitude of the carrier is varied proportional to that of the message signal.

Let m (t) be the base-band signal, m (t) ←→ M (ω) and c (t) be the carrier, $c(t) = A_c \cos(\omega_c t)$. fc is chosen such that fc >> W, where W is the maximum frequency component of m(t). The amplitude modulated signal is given by

$$s(t) = A_c [1 + k_a m(t)] \cos(\omega_c t)$$

Fourier Transform on both sides of the above equation

$$S(\omega) = \pi A_c/2\, (\delta(\omega - \omega_c) + \delta(\omega + \omega_c)) + k_a A_c/ 2\, (M(\omega - \omega_c) + M(\omega + \omega_c))$$

k_a is a constant called amplitude sensitivity.

$k_a m(t) < 1$ and it indicates percentage modulation.

Single Tone Modulation:

Consider a modulating wave m(t) that consists of a single tone or single frequency component given by

$$m(t) = A_m \cos(2\pi f_m t) \quad \text{.....................(1)}$$

where A_m is peak amplitude of the sinusoidal modulating wave

f_m is the frequency of the sinusoidal modulating wave

Let A_c be the peak amplitude and f_c be the frequency of the high frequency carrier signal. Then the corresponding single-tone AM wave is given by

$$s(t) = A_c[1 + m\cos(2\pi f_m t)]Cos(2\pi f_c t) \quad \text{.....................(2)}$$

Let A_{max} and A_{min} denote the maximum and minimum values of the envelope of the modulated wave. Then from the above equation (2.12), we get

$$\frac{A_{max}}{A_{min}} = \frac{A_c(1+m)}{A_c(1-m)}$$

$$m = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

Expanding the equation (2), we get

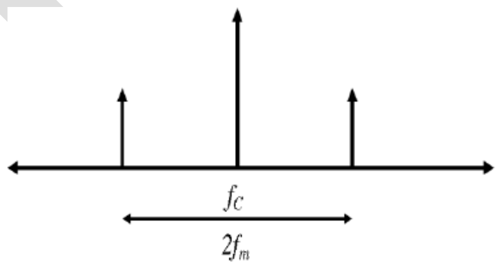
$$s(t) = A_c \cos(2\pi f_c t) + \frac{1}{2} m A_c \cos[2\pi(f_c + f_m)t] + \frac{1}{2} m A_c \cos[2\pi(f_c - f_m)t]$$

The Fourier transform of $s(t)$ is obtained as follows.

$$s(f) = \frac{1}{2} A_c[\delta(f - f_c) + \delta(f + f_c)] + \frac{1}{4} m A_c[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)]$$

$$+ \frac{1}{4} m A_c[\delta(f - f_c + f_m) + \delta(f + f_c - f_m)]$$

Thus the spectrum of an AM wave, for the special case of sinusoidal modulation consists of delta functions at $\pm f_c$, $f_c \pm f_m$, and $-f_c \pm f_m$. The spectrum for positive frequencies is as shown in figure



Frequency Domain characteristics of single tone AM

Power relations in AM waves:

Consider the expression for single tone/sinusoidal AM wave

$$s(t) = A_c Cos(2\pi f_c t) + \frac{1}{2} m A_c Cos[2\pi(f_c + f_m)t] + \frac{1}{2} m A_c Cos[2\pi(f_c - f_m)t] \quad \text{.................(1)}$$

This expression contains three components. They are carrier component, upper side band and lower side band. Therefore Average power of the AM wave is sum of these three components.

Therefore the total power in the amplitude modulated wave is given by

$$Pt = \frac{V^2{}_{car}}{R} + \frac{V^2{}_{LSB}}{R} + \frac{V^2{}_{USB}}{R} \quad \text{.......................(2)}$$

Where all the voltages are rms values and R is the resistance, in which the power is dissipated.

$$P_C = \frac{V^2{}_{car}}{R} = \frac{\left(A_c/\sqrt{2}\right)^2}{R} = \frac{A_c{}^2}{2R}$$

$$P_{LSB} = \frac{V_{LSB}{}^2}{R} = \left(\frac{mA_c}{2\sqrt{2}}\right)^2 \frac{1}{R} = \frac{m^2 A_c{}^2}{8R} = \frac{m^2}{4} P_c$$

$$P_{USB} = \frac{V_{USB}{}^2}{R} = \left(\frac{mA_c}{2\sqrt{2}}\right)^2 \frac{1}{R} = \frac{m^2 A_c{}^2}{8R} = \frac{m^2}{4} P_c$$

Therefore total average power is given by

$$P_t = P_c + P_{LSB} + P_{USB}$$

$$P_t = P_c + \frac{m^2}{4} P_c + \frac{m^2}{4} P_c$$

$$P_t = P_c \left(1 + \frac{m^2}{4} + \frac{m^2}{4}\right)$$

$$P_t = P_c \left(1 + \frac{m^2}{2}\right) \quad \text{..............................(3)}$$

The ratio of total side band power to the total power in the modulated wave is given by

$$\frac{P_{SB}}{P_t} = \frac{P_c(m^2/2)}{P_c(1+m^2/2)}$$

$$\frac{P_{SB}}{P_t} = \frac{m^2}{2+m^2} \qquad \text{.........................(4)}$$

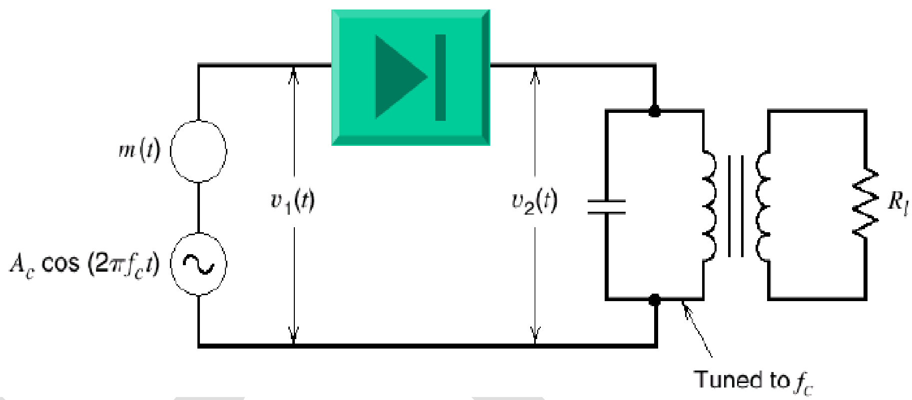
This ratio is called the efficiency of AM system

Generation of AM waves:

Two basic amplitude modulation principles are discussed. They are square law modulation and switching modulator.

Switching Modulator

Consider a semiconductor diode used as an ideal switch to which the carrier signal $c(t) = A_c \cos(2\pi f_c t)$ and information signal $m(t)$ are applied simultaneously as shown figure



Switching Modulator

The total input for the diode at any instant is given by

$$v_1 = c(t) + m(t)$$
$$v_1 = A_c \cos 2\pi f_c t + m(t)$$

When the peak amplitude of c(t) is maintained more than that of information signal, the operation is assumed to be dependent on only c(t) irrespective of m(t).

When c(t) is positive, v2=v1since the diode is forward biased. Similarly, when c(t) is negative, v2=0 since diode is reverse biased. Based upon above operation, switching response of the diode is periodic rectangular wave with an amplitude unity and is given by

$$p(t) = \frac{1}{2} + \frac{1}{\pi}\sum_{n=-\infty}^{\infty} \frac{(-1)^{n-1}}{2n-1}\cos(2\pi f_c t(2n-1))$$

$$p(t) = \frac{1}{2} + \underset{n=0,1}{\frac{2}{\pi}}\cos(2\pi f_c t) - \underset{n=-1,+2}{\frac{2}{3\pi}\cos(6\pi f_c t)} + -$$

Therefore the diode response V_o is a product of switching response $p(t)$ and input v_1.

$$v_2 = v_1 * p(t)$$

$$V_2 = [A_c \cos 2\pi f_c t + m(t)]\left[\frac{1}{2} + \frac{2}{\pi}\cos 2\pi f_c t - \frac{2}{3\pi}\cos 6\pi f_c t + - + -\right]$$

Applying the Fourier Transform, we get

$$V_2(f) = \frac{A_c}{4}[\delta(f - f_c) + \delta(f + f_c)] + \frac{M(f)}{2} + \frac{A_c}{\pi}\delta(f)$$
$$+ \frac{A_c}{2\pi}[\delta(f - 2f_c) + \delta(f + 2f_c)] + \frac{1}{\pi}[M(f - f_c) + M(f + f_c)]$$
$$- \frac{A_c}{6\pi}[\delta(f - 4f_c) + \delta(f + 4f_c)] - \frac{A_c}{3\pi}[\delta(f - 2f_c) + \delta(f + 2f_c)]$$
$$- \frac{1}{3\pi}[M(f - 3f_c) + M(f + f_c)]$$

The diode output v_2 consists of

a dc component at $f = 0$.

Information signal ranging from 0 to w Hz and infinite number of frequency bands centered at f, $2f_c$, $3f_c$, $4f_c$, ---------

The required AM signal centred at fc can be separated using band pass filter. The lower cut off-frequency for the band pass filter should be between w and fc-w and the upper cut-off frequency between fc+w and 2fc. The filter output is given by the equation

$$S(t) = \frac{A_c}{2}\left[1 + \frac{4}{\pi}\frac{m(t)}{A_c}\right]\cos 2\pi f_c t$$

For a single tone information, let $m(t) = A_m \cos(2\pi f_m t)$

$$S(t) = \frac{A_c}{2}\left[1 + \frac{4}{\pi}\frac{A_m}{A_c}\cos 2\pi f_m t\right]\cos 2\pi f_c t$$

Therefore modulation index, $m = \frac{4}{\pi}\frac{A_m}{A_c}$

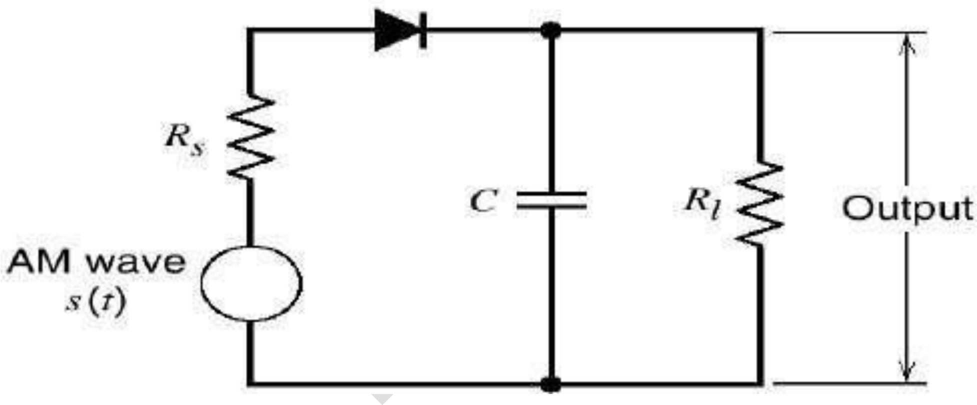
The output AM signal is free from distortions and attenuations only when $fc\text{-}w > w$ or $fc > 2w$.

Detection of AM waves

Demodulation is the process of recovering the information signal (base band) from the incoming modulated signal at the receiver. There are two methods, they are Square law Detector and Envelope Detector

Envelope Detector

It is a simple and highly effective system. This method is used in most of the commercial AM radio receivers. An envelope detector is as shown below.



Envelope Detector

During the positive half cycles of the input signals, the diode D is forward biased and the capacitor C charges up rapidly to the peak of the input signal. When the input signal falls below this value, the diode becomes reverse biased and the capacitor C discharges through the load resistor RL.

The discharge process continues until the next positive half cycle. When the input signal becomes greater than the voltage across the capacitor, the diode conducts again and the process is repeated.

The charge time constant (rf+Rs)C must be short compared with the carrier period, the capacitor charges rapidly and there by follows the applied voltage up to the positive peak when the diode is conducting.That is the charging time constant shall satisfy the condition,

$$(r_f + R_s)C << \frac{1}{f_c}$$

On the other hand, the discharging time-constant R_LC must be long enough to ensure that the capacitor discharges slowly through the load resistor R_L between the positive peaks of the carrier wave, but not so long that the capacitor voltage will not discharge at the maximum rate of change of the modulating wave.

That is the discharge time constant shall satisfy the condition,

$$\frac{1}{f_c} << R_LC << \frac{1}{W}$$

Where 'W' is band width of the message signal. The result is that the capacitor voltage or detector output is nearly the same as the envelope of AM wave.

Advantages and Disadvantages of AM:

Advantages of AM:

- Generation and demodulation of AM wave are easy.
- AM systems are cost effective and easy to build.

Disadvantages:

- AM contains unwanted carrier component; hence it requires more transmission power.
- The transmission bandwidth is equal to twice the message bandwidth.

To overcome these limitations, the conventional AM system is modified at the cost of increased system complexity. Therefore, three types of modified AM systems are discussed.

DSBSC (Double Side Band Suppressed Carrier) modulation:

In DSBC modulation, the modulated wave consists of only the upper and lower side bands. Transmitted power is saved through the suppression of the carrier wave, but the channel bandwidth requirement is the same as before.

SSBSC (Single Side Band Suppressed Carrier) modulation: The SSBSC modulated wave consists of only the upper side band or lower side band. SSBSC is suited for transmission of voice signals. It is an optimum form of modulation in that it requires the minimum transmission power and minimum channel band width. Disadvantage is increased cost and complexity.

VSB (Vestigial Side Band) modulation: In VSB, one side band is completely passed and just a trace or vestige of the other side band is retained. The required channel bandwidth is therefore in excess of the message bandwidth by an amount equal to the width of the vestigial side band. This method is suitable for the transmission of wide band signals.

DSB-SC MODULATION

DSB-SC Time domain and Frequency domain Description:

DSBSC modulators make use of the multiplying action in which the modulating signal multiplies the carrier wave. In this system, the carrier component is eliminated and both upper and lower side bands are transmitted. As the carrier component is suppressed, the power required for transmission is less than that of AM.

If $m(t)$ is the message signal and $c(t) = A_c \cos(2\pi f_c t)$ is the carrier signal, then DSBSC modulated wave $s(t)$ is given by

$$s(t) = c(t)\, m(t)$$

$$s(t) = A_c \cos(2\pi f_c t) m(t)$$

Consequently, the modulated signal s(t) under goes a phase reversal , whenever the message signal m(t) crosses zero as shown below.

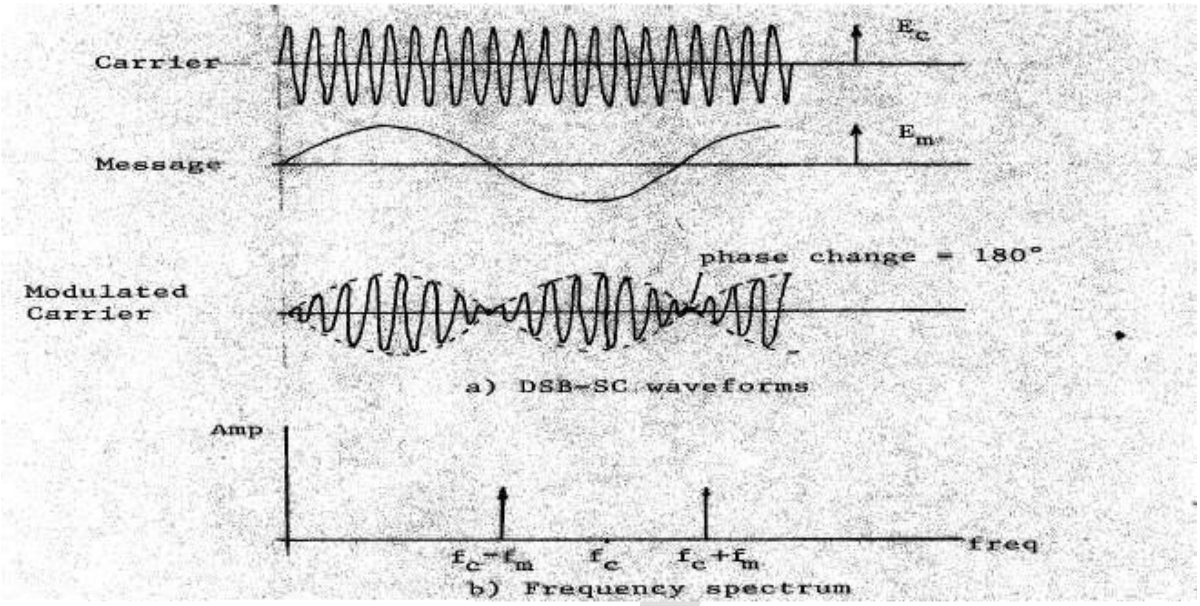


Fig.1. (a) DSB-SC waveform (b) DSB-SC Frequency Spectrum

The envelope of a DSBSC modulated signal is therefore different from the message signal and the Fourier transform of s(t) is given by

$$S(f) = \frac{A_c}{2}\left[M(f - f_c) + M(f + f_c)\right]$$

For the case when base band signal *m(t)* is limited to the interval $-W<f<W$ as shown in figure below, we find that the spectrum *S(f)* of the DSBSC wave *s(t)* is as illustrated below. Except for a change in scaling factor, the modulation process simply translates the spectrum of the base band signal by f_c. The transmission bandwidth required by DSBSC modulation is the same as that for AM.

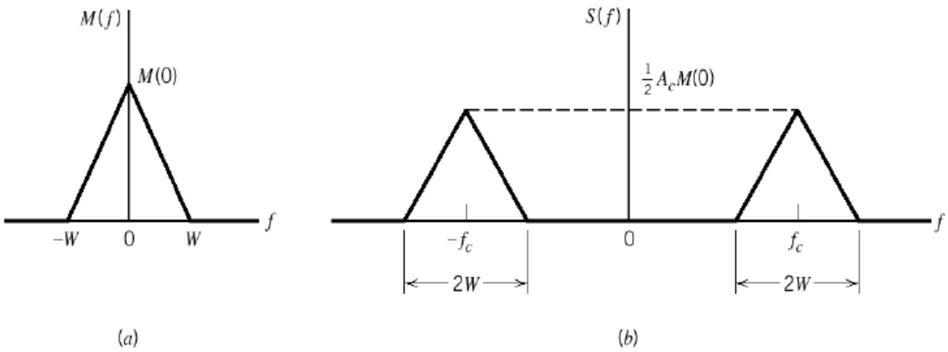


Figure: Message and the corresponding DSBSC spectrum

Generation of DSBSC Waves:

Balanced Modulator (Product Modulator)

A balanced modulator consists of two standard amplitude modulators arranged in a balanced configuration so as to suppress the carrier wave as shown in the following block diagram. It is assumed that the AM modulators are identical, except for the sign reversal of the modulating wave applied to the input of one of them. Thus, the output of the two modulators may be expressed as,

$$s_1(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$
$$s_2(t) = A_c[1 - k_a m(t)]\cos(2\pi f_c t)$$

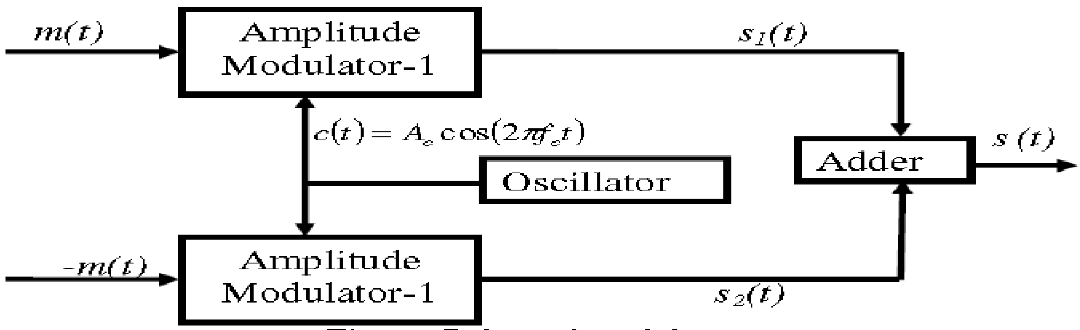


Fig 3 : Balanced modulator

Subtracting $s_2(t)$ from $s_1(t)$,

$$s(t) = s_1(t) - s_2(t)$$
$$s(t) = 2k_a m(t) A_c \cos(2\pi f_c t)$$

Hence, except for the scaling factor 2ka, the balanced modulator output is equal to the product of the modulating wave and the carrier.

Detection of DSB-SC waves:

Coherent Detection:

The message signal m(t) can be uniquely recovered from a DSBSC wave s(t) by first multiplying s(t) with a locally generated sinusoidal wave and then low pass filtering the product as shown.

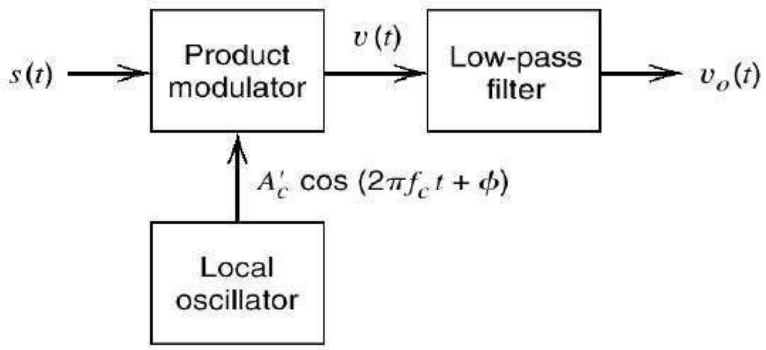


Fig.5 : Coherent detector

It is assumed that the local oscillator signal is exactly coherent or synchronized, in both frequency and phase, with the carrier wave c(t) used in the product modulator to generate s(t). This method of demodulation is known as coherent detection or synchronous detection.

Let $A_c^1 \cos(2\pi f_c t + \phi)$ be the local oscillator signal, and $s(t) = A_c \cos(2\pi f_c t) m(t)$ be the DSBSC wave. Then the product modulator output $v(t)$ is given by

$$v(t) = A_c A_c^1 \cos(2\pi f_c t)\cos(2\pi f_c t + \phi) m(t)$$

$$v(t) = \frac{A_c A_c^1}{4}\cos(4\pi f_c t + \phi) m(t) + \frac{A_c A_c^1}{2}\cos(\phi) m(t)$$

The first term in the above expression represents a DSBSC modulated signal with a carrier frequency $2f_c$, and the second term represents the scaled version of message signal. Assuming that the message signal is band limited to the interval $-w < f < w$, the spectrum of $v(t)$ is plotted as shown below.

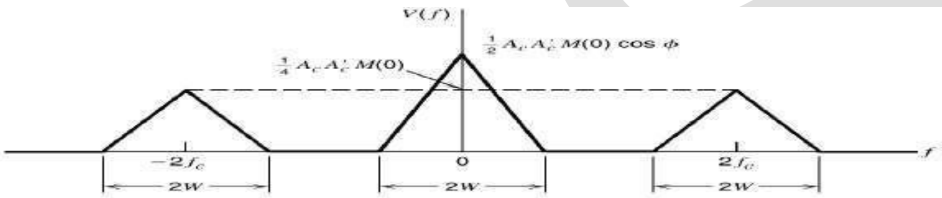


Fig.6.Spectrum of output of the product modulator

From the spectrum, it is clear that the unwanted component (first term in the expression) can be removed by the low-pass filter, provided that the cut-off frequency of the filter is greater than W but less than 2fc-W. The filter output is given by

$$v_o(t) = \frac{A_c A_c^1}{2}\cos(\phi) m(t)$$

The demodulated signal vo(t) is therefore proportional to m(t) when the phase error ϕ is constant.

Costas Receiver (Costas Loop):

Costas receiver is a synchronous receiver system, suitable for demodulating DSBSC waves. It consists of two coherent detectors supplied with the same input signal, that is the incoming DSBSC wave $s(t) = A_c \cos(2\pi f_c t) m(t)$ but with individual local oscillator signals that are in phase quadrature with respect to each other as shown below.

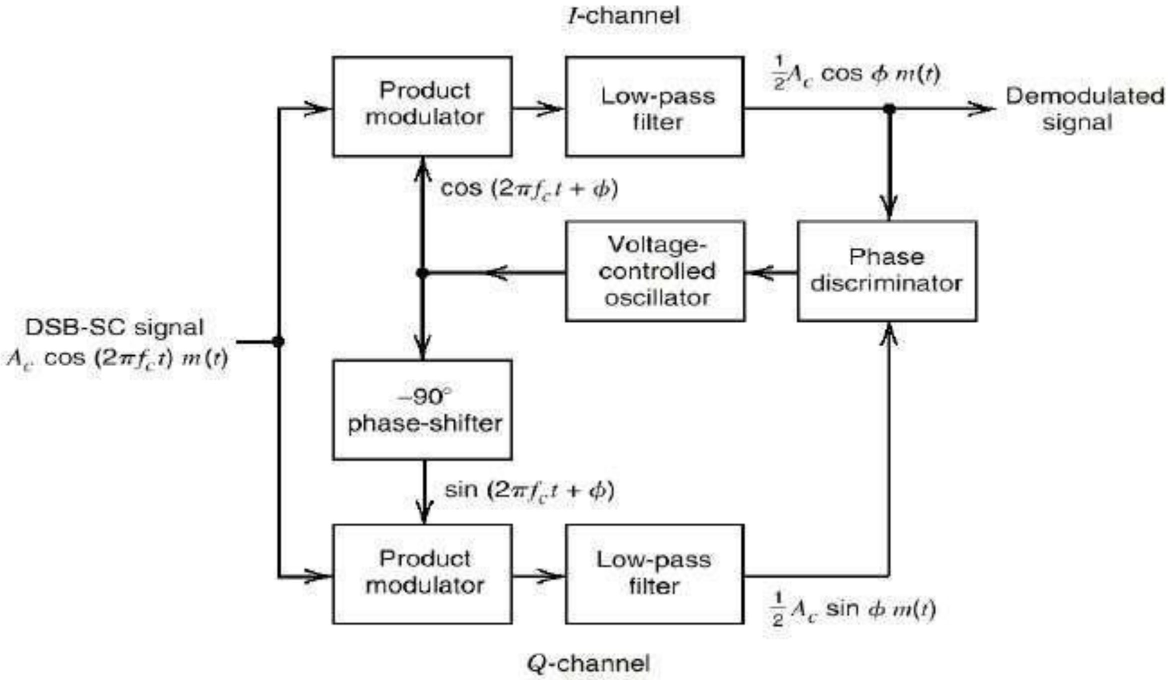


Fig.7. Costas Receiver

The frequency of the local oscillator is adjusted to be the same as the carrier frequency fc. The detector in the upper path is referred to as the in-phase coherent detector or I-channel, and that in the lower path is referred to as the quadrature-phase coherent detector or Q-channel.

These two detectors are coupled together to form a negative feedback system designed in such a way as to maintain the local oscillator synchronous with the carrier wave. Suppose the local oscillator signal is of the same phase as the carrier c(t) = Accos(2πfct) wave used to generate the incoming DSBSC wave. Then we find that the I-channel output contains the desired demodulated signal m(t), whereas the Q-channel output is zero due to quadrature null effect of the Q-channel. Suppose that the local oscillator phase drifts from its proper value by a small angle ϕ radians. The I-channel output will remain essentially unchanged, but there will be some signal appearing at the Q-channel output, which is proportional to $\sin(\phi) \approx \phi$ for small ϕ.

This Q-channel output will have same polarity as the I-channel output for one direction of local oscillator phase drift and opposite polarity for the opposite direction of local oscillator phase drift. Thus, by combining the I-channel and Q-channel outputs in a phase discriminator (which consists of a multiplier followed by a LPF), a dc control signal is obtained that automatically corrects for the local phase errors in the voltage-controlled oscillator.

Introduction of SSB-SC

Standard AM and DSBSC require transmission bandwidth equal to twice the message bandwidth. In both the cases spectrum contains two side bands of width W Hz, each. But the upper and lower sides are uniquely related to each other by the virtue of their symmetry about the carrier frequency. That is, given the amplitude and phase spectra of either side band, the other can be uniquely determined. Thus if only one side band is transmitted, and if both the carrier and the other side band are suppressed at the transmitter, no information is lost. This kind of modulation is called SSBSC and spectral comparison between DSBSC and SSBSC is shown in the figures 1 and 2.

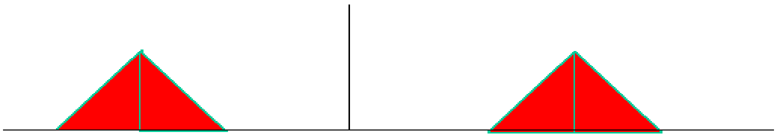


Figure.1 : Spectrum of the DSBSC wave

Figure .2 : Spectrum of the SSBSC wave

Frequency Domain Description

Consider a message signal *m(t)* with a spectrum *M(f)* band limited to the interval $-w < f < w$ as shown in figure 3 , the DSBSC wave obtained by multiplexing *m(t)* by the carrier wave $c(t) = A_c \cos(2\pi f_c t)$and is also shown, in figure 4 . The upper side band is represented in duplicate by the frequencies above f_c and those below $-f_c$, and when only upper

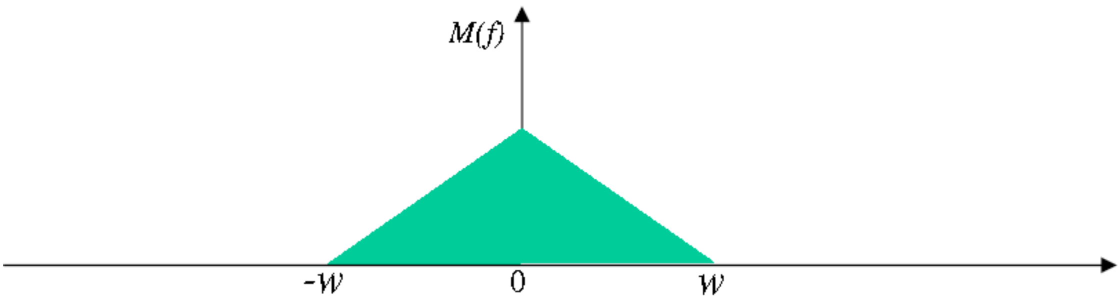


Figure 3. : Spectrum of message wave

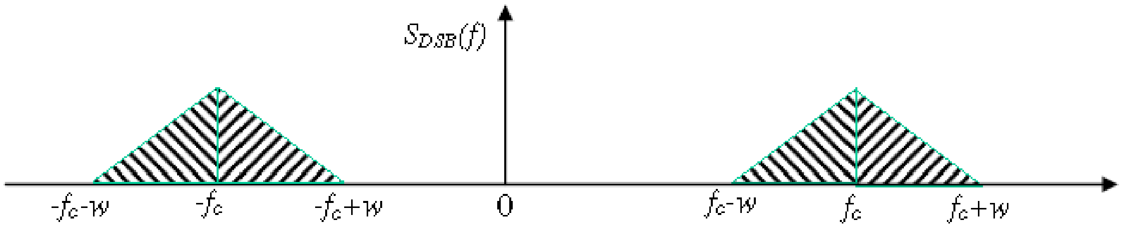


Figure .4 : Spectrum of DSBSC wave

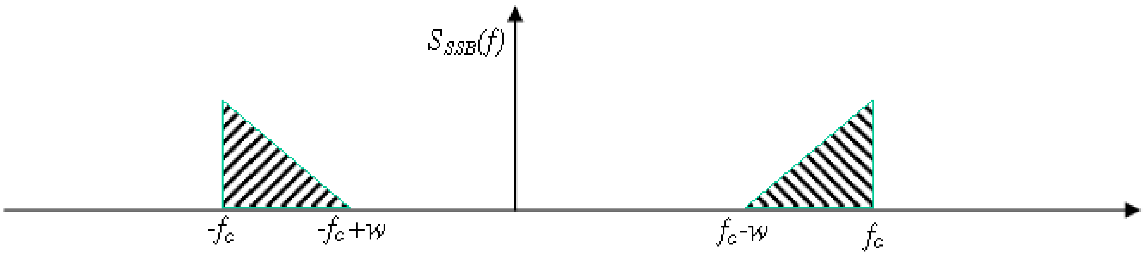


Figure.5 : Spectrum of SSBSC-LSB wave

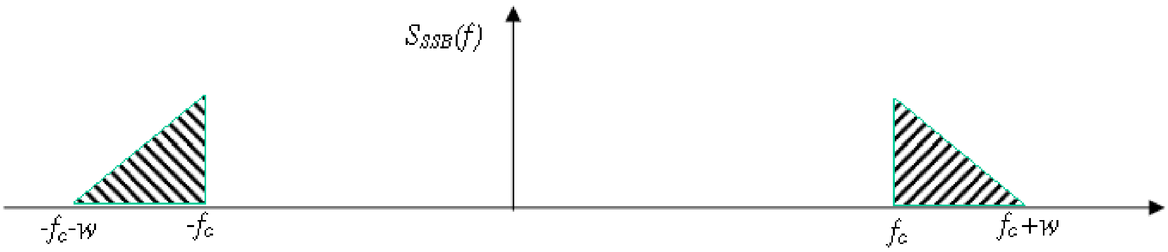


Figure .6 : Spectrum of SSBSC-USB wave

side band is transmitted; the resulting SSB modulated wave has the spectrum shown in figure 6. Similarly, the lower side band is represented in duplicate by the frequencies below fc and those above -fc and when only the lower side band is transmitted, the spectrum of the corresponding SSB modulated wave shown in figure 5.Thus the essential function of the SSB modulation is to translate the spectrum of the modulating wave, either with or without inversion, to a new location in the frequency domain. The advantage of SSB modulation is reduced bandwidth and the elimination of high power carrier wave. The main disadvantage is the cost and complexity of its implementation.

Generation of SSB wave:

Frequency discrimination method

Consider the generation of SSB modulated signal containing the upper side band only. From a practical point of view, the most severe requirement of SSB generation arises from the unwanted sideband, the nearest component of which is separated from the desired side band by twice the lowest frequency component of the message signal. It implies that, for the generation of an SSB wave to be possible, the message spectrum must have an energy gap centered at the origin as shown in figure 7. This requirement is naturally satisfied by voice signals, whose energy gap is about 600Hz wide.

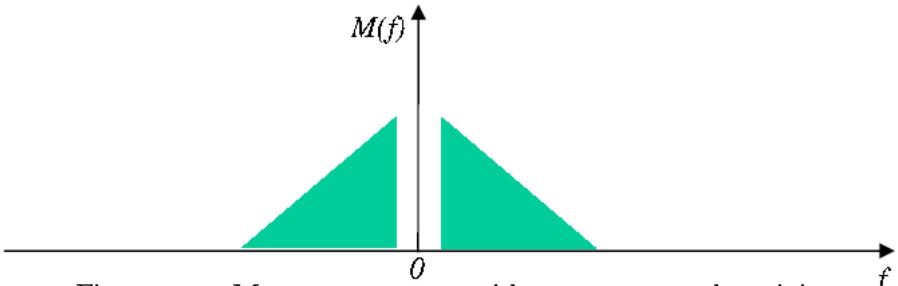


Figure .7 : Message spectrum with energy gap at the origin

The frequency discrimination or filter method of SSB generation consists of a product modulator, which produces DSBSC signal and a band-pass filter to extract the desired side band and reject the other and is shown in the figure 8.

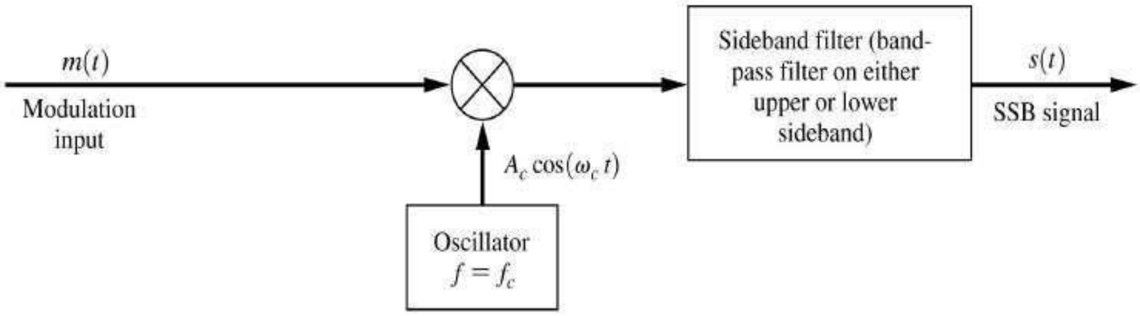


Figure .8 : Frequency discriminator to generate SSBSC wave

Application of this method requires that the message signal satisfies two conditions:
1. The message signal m(t) has no low-frequency content. Example: speech, audio, music.
2. The highest frequency component W of the message signal m(t) is much less than the carrier frequency fc.

Then, under these conditions, the desired side band will appear in a non-overlapping interval in the spectrum in such a way that it may be selected by an appropriate filter.

In designing the band pass filter, the following requirements should be satisfied:

1. The pass band of the filter occupies the same frequency range as the spectrum of the desired SSB modulated wave.

2. The width of the guard band of the filter, separating the pass band from the stop band, where the unwanted sideband of the filter input lies, is twice the lowest frequency component of the message signal.

When it is necessary to generate an SSB modulated wave occupying a frequency band that is much higher than that of the message signal, it becomes very difficult to design an appropriate filter that will pass the desired side band and reject the other. In such a situation it is necessary to resort to a multiple-modulation process so as to ease the filtering requirement. This approach is illustrated in the following figure 9 involving two stages of modulation.

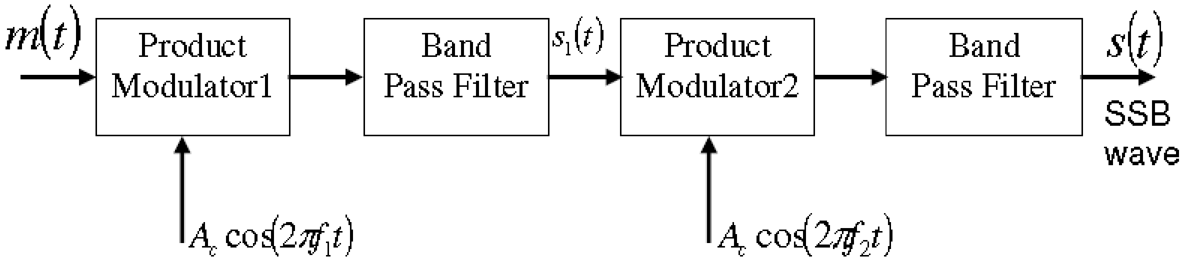


Figure .9 : Two stage frequency discriminator

The SSB modulated wave at the first filter output is used as the modulating wave for the second product modulator, which produces a DSBSC modulated wave with a spectrum that is symmetrically spaced about the second carrier frequency f2. The frequency separation between the side bands of this DSBSC modulated wave is effectively twice the first carrier frequency f1, thereby permitting the second filter to remove the unwanted side band.

Time Domain Description:

The time domain description of an SSB wave s(t) in the canonical form is given by the equation 1.

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t) \quad \text{-------------------- (1)}$$

where $S_I(t)$ is the in-phase component of the SSB wave and $S_Q(t)$ is its quadrature component. The in-phase component $S_I(t)$ except for a scaling factor, may be derived from $S(t)$ by first multiplying $S(t)$ by $\cos(2\pi f_c t)$ and then passing the product through a low-pass filter. Similarly, the quadrature component $S_Q(t)$, except for a scaling factor, may be derived from $s(t)$ by first multiplying $s(t)$ by $\sin(2\pi f_c t)$ and then passing the product through an identical filter.

The Fourier transformation of $S_I(t)$ and $S_Q(t)$ are related to that of SSB wave as follows, respectively.

$$S_I(f) = \begin{cases} S(f - f_c) + S(f + f_c), -w \le f \le w \\ 0, elsewhere \end{cases} \quad \text{------------------- (2)}$$

$$S_Q(f)=\begin{cases} j[S(f-f_c)-S(f+f_c)], -w \le f \le w \\ 0, elsewhere \end{cases} \quad \text{------------------ (3)}$$

where $-w < f < w$ defines the frequency band occupied by the message signal $m(t)$.

Consider the SSB wave that is obtained by transmitting only the upper side band, shown in figure 10 . Two frequency shifted spectras $S(f-f_c)$ and $S(f+f_c)$ are shown in figure 11 and figure 12 respectively. Therefore, from equations 2 and 3 , it follows that the corresponding spectra of the in- phase component $S_I(t)$ and the quadrature component $S_Q(t)$ are as shown in figure 13 and 14 respectively.

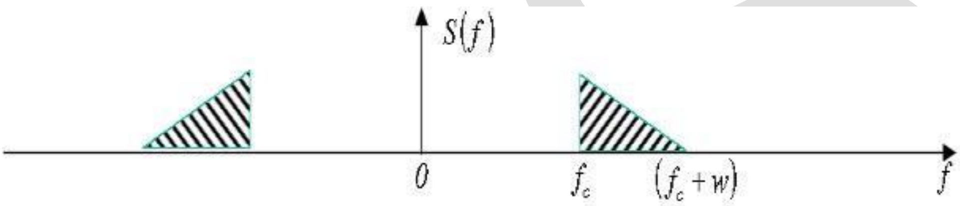


Figure 10 : Spectrum of SSBSC-USB

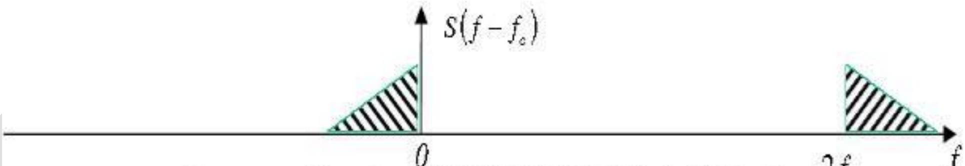


Figure 11 : Spectrum of SSBSC-USB shifted right by f_c

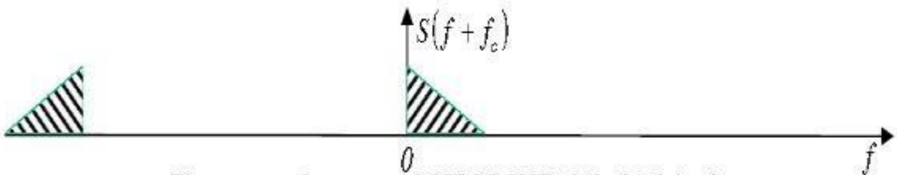


Figure 12 : Spectrum of SSBSC-USB shifted left by f_c

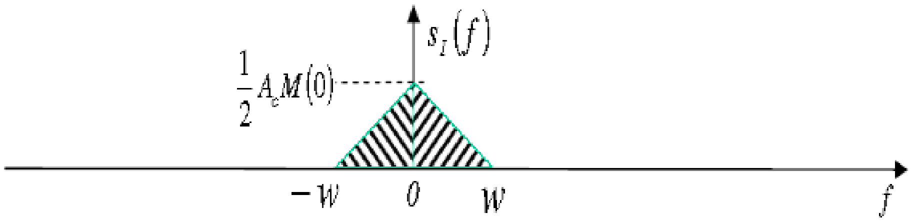


Figure 13 : Spectrum of in-phase component of SSBSC-USB

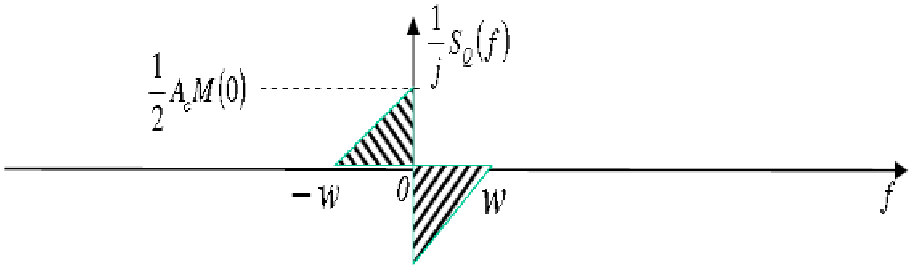


Figure 14 : Spectrum of quadrature component of SSBSC-USB

From the figure 13 , it is found that

$$S_I(f)=\frac{1}{2}A_cM(f)$$

where $M(f)$ is the Fourier transform of the message signal $m(t)$. Accordingly in-phase component $S_I(t)$ is defined by equation 4

$$s_I(t)=\frac{1}{2}A_cm(t) \qquad \text{------------------- (4)}$$

Now on the basis of figure14 , it is found that

$$S_Q(f)=\begin{cases}\frac{-j}{2}A_cM(f), f>0\\ 0, f=0\\ \frac{j}{2}A_cM(f), f<0\end{cases}$$

$$S_Q(f)=\frac{-j}{2}A_c\operatorname{sgn}(f)M(f) \qquad \text{------------------- (5)}$$

where $\operatorname{sgn}(f)$ is the Signum function.

But from the discussions on Hilbert transforms, it is shown that

$$-j\,\mathrm{sgn}(f)M(f)=\hat{M}(f) \qquad \text{------------------ (6)}$$

where $\hat{M}(f)$ is the Fourier transform of the Hilbert transform of *m(t)*. Hence the substituting equation (6) in (5), we get

$$S_Q(f)=\frac{1}{2}A_c\hat{M}(f) \qquad \text{------------------------ (7)}$$

Therefore quadrature component $s_Q(t)$ is defined by equation 8

$$s_Q(t)=\frac{1}{2}A_c\hat{m}(t) \qquad \text{------------------------- (8)}$$

Therefore substituting equations (4) and (8) in equation in (1), we find that canonical representation of an SSB wave *s(t)* obtained by transmitting only the upper side band is given by the equation 9

$$s_U(t)=\frac{1}{2}A_c m(t)\cos(2\pi f_c t)-\frac{1}{2}A_c\hat{m}(t)\sin(2\pi f_c t) \qquad \text{------------- (9)}$$

Following the same procedure, we can find the canonical representation for an SSB wave
s(t) obtained by transmitting only the lower side band is given by

$$s_L(t)=\frac{1}{2}A_c m(t)\cos(2\pi f_c t)+\frac{1}{2}A_c\hat{m}(t)\sin(2\pi f_c t) \qquad \text{-------------- (10)}$$

Phase discrimination method for generating SSB wave:

Time domain description of SSB modulation leads to another method of SSB generation using the equations 9 or 10. The block diagram of phase discriminator is as shown in figure 15.

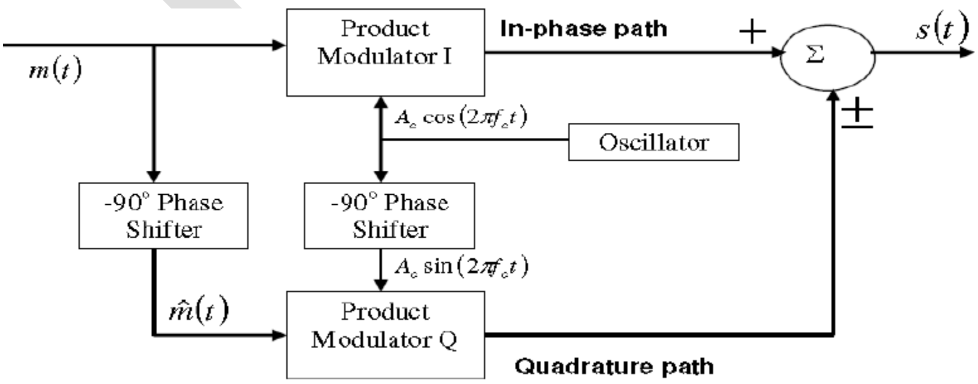


Figure 15 : Block diagram of phase discriminator

The phase discriminator consists of two product modulators I and Q, supplied with carrier waves in-phase quadrature to each other. The incoming base band signal m(t) is applied to product modulator I, producing a DSBSC modulated wave that contains reference phase sidebands symmetrically spaced about carrier frequency fc.

The Hilbert transform m^ (t) of m (t) is applied to product modulator Q, producing a DSBSC modulated that contains side bands having identical amplitude spectra to those of modulator I, but with phase spectra such that vector addition or subtraction of the two modulator outputs results in cancellation of one set of side bands and reinforcement of the other set.

The use of a plus sign at the summing junction yields an SSB wave with only the lower side band, whereas the use of a minus sign yields an SSB wave with only the upper side band. This modulator circuit is called Hartley modulator.

Demodulation of SSB Waves:

Demodulation of SSBSC wave using coherent detection is as shown in 16 . The SSB wave *s(t)* together with a locally generated carrier $c(t) = A_c{}^1 \cos(2\pi f'_c t + \phi)$ is applied to a product modulator and then low-pass filtering of the modulator output yields the message signal.

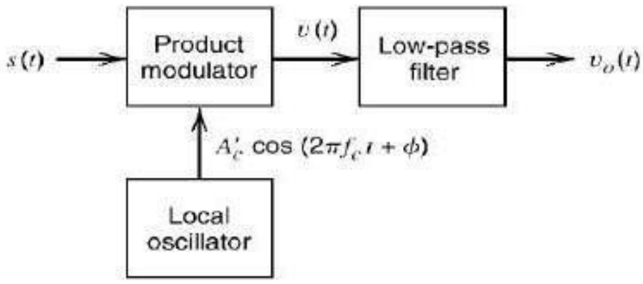


Figure 16 : Block diagram of coherent detector for SSBSC

The product modulator output *v(t)* is given by

$$v(t) = A_c{}^1 \cos(2\pi f_c t + \phi)s(t) \quad , \quad \text{Put } \phi = 0$$

$$v(t) = \frac{1}{2} A_c \cos(2\pi f_c t)[m(t)\cos(2\pi f_c t) \pm \hat{m}(t)\sin(2\pi f_c t)]$$

$$v(t) = \frac{1}{4} A_c m(t) + \frac{1}{4} A_c [m(t)\cos(4\pi f_c t) \pm \hat{m}(t)\sin(4\pi f_c t)] \quad \text{..............(1)}$$

The first term in the above equation 1 is desired message signal. The other term represents an SSB wave with a carrier frequency of $2f_c$ as such; it is an unwanted component, which is removed by low-pass filter.

Introduction to Vestigial Side Band Modulation

Vestigial sideband is a type of Amplitude modulation in which one side band is completely passed along with trace or tail or vestige of the other side band. VSB is a compromise between SSB and DSBSC modulation. In SSB, we send only one side band, the Bandwidth required to send SSB wave is w. SSB is not appropriate way of modulation when the message signal contains significant components at extremely low frequencies. To overcome this VSB is used.

Vestigial Side Band (VSB) modulation is another form of an amplitude-modulated signal in which a part of the unwanted sideband (called as vestige, hence the name vestigial sideband) is allowed to appear at the output of VSB transmission system.

The AM signal is passed through a sideband filter before the transmission of SSB signal. The design of sideband filter can be simplified to a greater extent if a part of the other sideband is also passed through it. However, in this process the bandwidth of VSB system is slightly increased.

Generation of VSB Modulated Signal

VSB signal is generated by first generating a DSB-SC signal and then passing it through a sideband filter which will pass the wanted sideband and a part of unwanted sideband. Thus, VSB is so called because a vestige is added to SSB spectrum.

The below figure depicts functional block diagram of generating VSB modulated signal

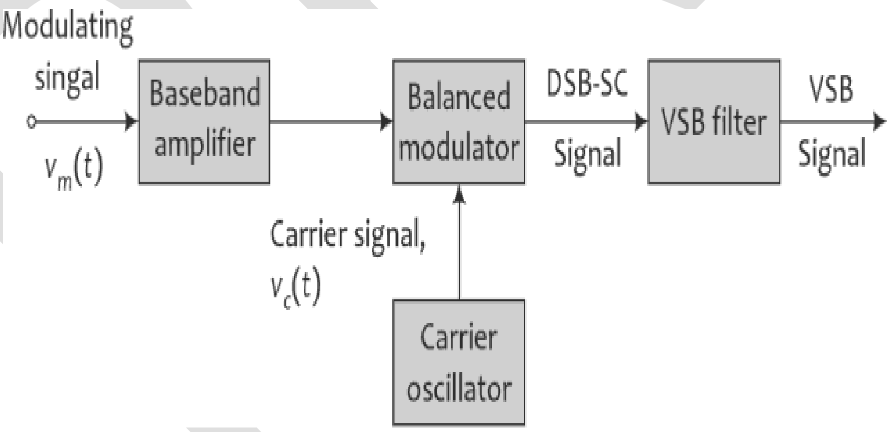


Figure: Generation of VSB Modulated Signal

A VSB-modulated signal is generated using the frequency discrimination method, in which firstly a DSB-SC modulated signal is generated and then passed through a sideband- suppression filter. This type of filter is a specially-designed bandpass filter that distinguishes VSB modulation from SSB modulation.the cutoff portion of the frequency response of this filter around the carrier frequency exhibits odd symmetry, that is, (fc-fv)≤|f|≤(fc+fv).

Accordingly, the bandwidth of the VSB signal is given as

BW=(fm+fv) Hz

Where fm is the bandwidth of the modulating signal or USB, and fv is the bandwidth of vestigial sideband (VSB)

Time domain description of VSB Signal

Mathematically, the VSB modulated signal can be described in the time-domain

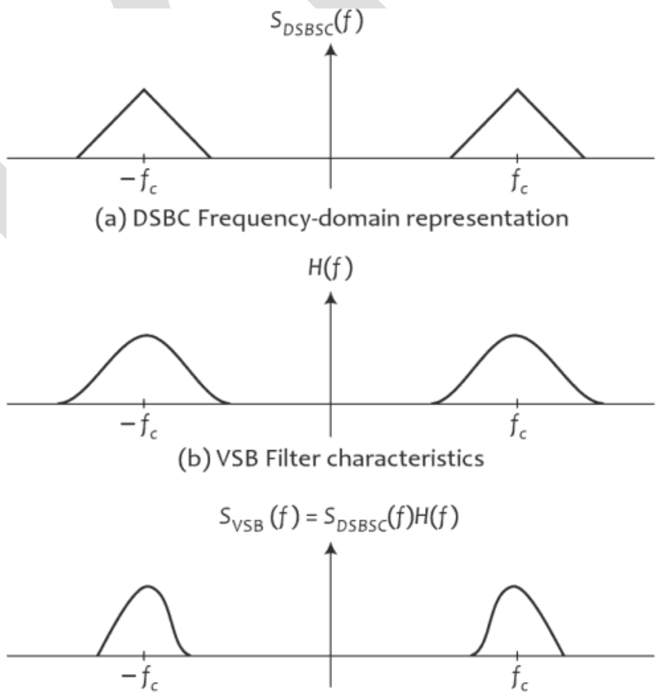
as s(t)= m(t) Ac cos(2πfct) ± mQ(t) Ac sin(2πfct)

where m(t) is the modulating signal, mQ(t) is the component of m(t) obtained by passing the message signal through a vestigial filter, Ac cos(2πfct) is the carrier signal, and Ac sin(2πfct) is the 90^{o} phase shift version of the carrier signal.

The ± sign in the expression corresponds to the transmission of a vestige of the upper-sideband and lower-sideband respectively. The Quadrature component is required to partially reduce power in one of the sidebands of the modulated wave s(t) and retain a vestige of the other sideband as required.

Frequency domain representation of VSB Signal

Since VSB modulated signal includes a vestige (or trace) of the second sideband, only a part of the second sideband is retained instead of completely eliminating it. Therefore, VSB signal can be generated from DSB signal followed by VSB filter which is a practical filter.



(a) DSBC Frequency-domain representation

(b) VSB Filter characteristics

(c) VSB Frequency-domain representation

The below figure shows the DSB signal spectrum, the VSB filter characteristics, and the resulting output VSB modulated signal spectrum.

Bandwidth Consideration in TV Signals

An important application of VSB modulation technique is in broadcast television. In commercial TV broadcasting system, there is a basic need to conserve bandwidth.

- The upper-sideband of the video carrier signal is transmitted upto 4MHz without any attenuation.
- The lower-sideband of the video carrier signal is transmitted without any attenuation over the range 0.75 MHz (Double side band transmission) and is entirely attenuated at
  1.25MHz (single sideband transmission) and the transition is made from one o another between 0.75MHz and 1.25 MHz (thus the name vestige sideband)
- The audio signal which accompanies the video signal is transmitted by frequency modulation method using a carrier signal located 4.5 MHz above the video-carrier signal.
- The audio signal is frequency modulated on a separate carrier signal with a frequency deviation of 25 KHz. With an audio bandwidth of 10 KHz, the deviation ratio is 2.5 and an FM bandwidth of approximately 70 KHz.
- The frequency range of 100 KHz is allowed on each side of the audio-carrier signal for the audio sidebands.
- One sideband of the video-modulated signal is attenuated so that it does not interfere with the lower- sideband of the audio carrier.

Advantages of VSB Modulation

VSB transmission system has several advantages which include

- Use of simple filter design
- Less bandwidth as compared to that of DSBSC signal
- As efficient as SSB
- Possibility of transmission of low frequency components of modulating signals

Facts to Know

VSB is mainly used as a standard modulation technique for transmission of video signals in TV signals in commercial television broadcasting because the modulating video signal has large bandwidth and high speed data transmission

Envelope detection of a VSB Wave plus Carrier

To make demodulation of VSB wave possible by an envelope detector at the receiving end it is necessary to transmit a sizeable carrier together with the modulated wave. The scaled expression of VSB wave by factor k_a with the carrier component $A_c \cos(2\pi f_c t)$ can be given by

$$s(t) = A_c \cos(2\pi f_c t) + \frac{A_c}{2} k_a m(t) \cos(2\pi f_c t) - \frac{A_c}{2} k_a m_Q(t) \sin(2\pi f_c t)$$

$$= A_c\left[1+\frac{k_a}{2} m(t)\right]\cos(2\pi f_c t) - \frac{A_c k_a}{2} m_Q(t) \sin(2\pi f_c t) \quad \text{.................(1)}$$

where k_a is the modulation index; it determines the percentage modulation.

When above signal s(t) is passed through the envelope detector, the detector output is given by,

$$a(t) = A_c\left[\left(1+\frac{k_a}{2} m(t)\right)^2 + \left(\frac{k_a}{2} m_Q(t)\right)^2\right]^{1/2}$$

$$= A_c\left[1+\frac{k_a}{2} m(t)\right]\left(1+\left[\frac{\frac{1}{2} k_a m_Q(t)}{1+\frac{1}{2} k_a m(t)}\right]^2\right)^{1/2} \quad \text{.................(2)}$$

The detector output is distorted by the quadrature component $m_Q(t)$ as indicated by equation (2).

Methods to reduce distortion

- Distortion can be reduced by reducing percentage modulation, k_a.
- Distortion can be reduced by reducing $m_Q(t)$ by increasing the width of the vestigial sideband.

Comparison of AM Techniques:

Sr. No.	Parameter	Standard AM	SSB	DSBSC	VSB
1	Power	High	Less	Medium	Less than DSBSC but greater than SSB
2	Bandwidth	$2f_m$	f_m	$2f_m$	$f_m < B_w < 2f_m$
3	Carrier supression	No	Yes	Yes	No
4	Receiver complexity	Simple	Complex	Complex	Simple
5	Application	Radio communication	Point to point communication preferred for long distance transmission.	Point to point communication	Television broadcasting
6	Modulation type	Non linear	Linear	Linear	Linear
7	Sideband suppression	No	One sided completely	No	One sideband suppressed partly
8	Transmission efficiency	Minimum	Maximum	Moderate	Moderate

Applications of different AM systems:

- Amplitude Modulation: AM radio, Short wave radio broadcast
- DSB-SC: Data Modems, Color TV's color signals.
- SSB: Telephone
- VSB: TV picture signals

Angle Modulation (Phase and Frequency Modulation)

Introduction

There are two forms of angle modulation that may be distinguished – phase modulation and frequency modulation

Basic Definitions: Phase Modulation (PM) and Frequency Modulation (FM)

Let $\theta_i(t)$ denote the angle of modulated sinusoidal carrier, which is a function of the message. The resulting angle-modulated wave is expressed as

$$s(t) = A_c\cos[\theta_i(t)] \ldots\ldots\ldots\ldots (1)$$

Where A_c is the carrier amplitude. A complete oscillation occurs whenever $\theta_i(t)$ changes by 2π radians. If $\theta_i(t)$ increases monotonically with time, the average frequency in Hz, over an interval from t to $t+\Delta t$, is given by

$$f_{\Delta t}(t) = \frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi\Delta t} \quad (2)$$

Thus the instantaneous frequency of the angle-modulated wave *s(t)* is defined as

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t)$$

$$f_i(t) = \lim_{\Delta t \to 0} \left[\frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi\Delta t}\right]$$

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} \ldots\ldots\ldots\ldots\ldots (3)$$

Thus, according to equation (1), the angle modulated wave s(t) is interpreted as a rotating Phasor of length Ac and angle $\theta_i(t)$. The angular velocity of such a Phasor is $d\theta_i(t)/dt$, in accordance with equ (3).In the simple case of an unmodulated carrier, the angle $\theta_i(t)$ is

$$\theta_i(t) = 2\pi f_c t + \phi_c$$

And the corresponding Phasor rotates with a constant angular velocity equal to $2\pi f_c$.The constant ϕ_c is the value of (t) at t=0.

There are an infinite number of ways in which the angle (t) may be varied in some manner with the baseband signal.

But the 2 commonly used methods are **Phase modulation** and **Frequency modulation.**

Phase Modulation (PM) is that form of angle modulation in which the angle (t) is varied linearly with the baseband signal m(t), as shown by

$$\theta_i(t) = 2\pi f_c t + k_p m(t) \ldots\ldots\ldots\ldots\ldots (4)$$

The term $2\pi f_c t$ represents the angle of the unmodulated carrier, and the constant k_p represents the ***phase sensitivity*** of the modulator, expressed in radians per volt. The phase-modulated wave *s(t)* is thus described in time domain by

$$s(t) = A_c \cos[2\pi f_c t + k_p m(t)] \ldots\ldots\ldots\ldots (5)$$

Frequency Modulation (FM) is that form of angle modulation in which the instantaneous frequency $f_i(t)$ is varied linearly with the baseband signal *m(t)*, as shown by

$$f_i(t) = f_c + k_f m(t) \ldots\ldots\ldots\ldots\ldots (6)$$

The term f_c represents the frequency of the unmodulated carrier, and the constant k_f represents the ***frequency sensitivity*** of the modulator, expressed in hertz per volt. Integrating equ.(6) with respect to time and multiplying the result by 2π, we get

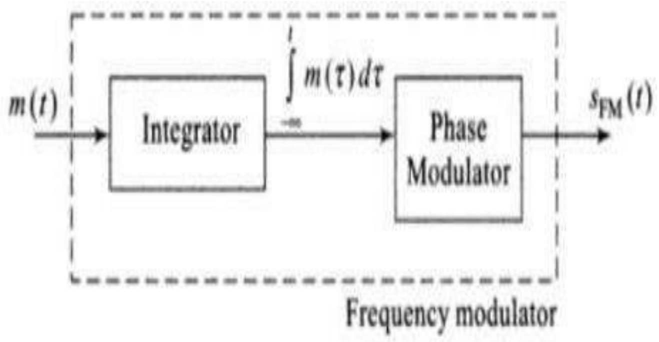
$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(t)\ dt \text{ .... } (7)$$

Where, for convenience it is assumed that the angle of the unmodulated carrier wave is zero at t=0. The frequency modulated wave is therefore described in the time domain by

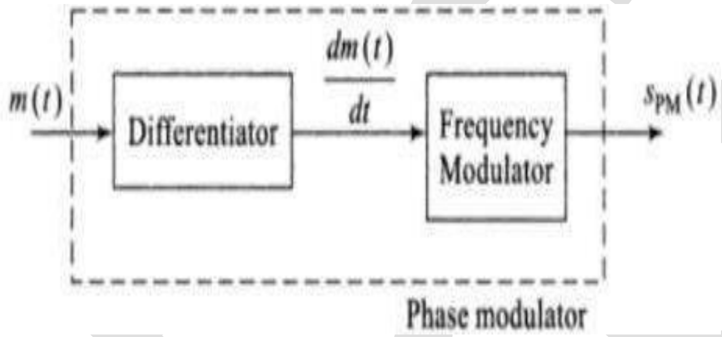
$$s(t) = A_c \cos\left[\, 2\pi f_c t + 2\pi k_f \int_0^t m(t)\ dt\right] \text{.............................} (8)$$

Relationship between PM and FM

Comparing equ (5) with (8) reveals that an FM wave may be regarded as a PM wave in which the modulating wave is $\int_0^t (t)t$ in place of $m(t)$.



A PM wave can be generated by first differentiating *m(t)* and then using the result as the input to a frequency modulator.



Thus, the properties of PM wave can be deduced from those of FM waves and vice versa

Single tone Frequency modulation

Consider a sinusoidal modulating wave defined by

$$m(t) = Am\cos(2\pi fmt) \ldots\ldots\ldots\ldots(1)$$

The instantaneous frequency of the resulting FM wave is

$$fi(t) = fc + kfAm\cos(2\pi fmt)$$

$$fi(t) = fc + \Delta f\cos(2\pi fmt) \ldots\ldots\ldots\ldots(2)$$

Where $$\Delta f = kfAm \ldots\ldots\ldots\ldots(3)$$

The quantity Δf is called the ***frequency deviation***, representing the maximum departure of the instantaneous frequency of the FM wave from the carrier frequency fc.

Fundamental characteristic of an FM wave is that the frequency deviation Δf is proportional to the amplitude of the modulating wave, and is independent of the modulation frequency.

Using equation (2), the angle (t) of the FM wave is obtained as

$$\theta i(t) = 2\pi \int_{0}^{t} fi(t)dt$$

$$\theta i(t) = 2\pi fct + \frac{\Delta f}{fm} sin(2\pi fmt) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The ratio of the frequency deviation *Δf to* the modulation frequency *fm* is commonly called the ***modulation index*** of the FM wave. Modulation index is denoted by $\boldsymbol{\beta}$ and is given as

$$Q = \frac{\Delta f}{fm} \ldots\ldots\ldots\ldots\ldots\ldots\ldots. (5)$$

And

$$\theta i(t) = 2\pi f ct + \text{Q}sin(2\pi f mt) \ldots\ldots\ldots\ldots\ldots (6)$$

In equation (6) the parameter β represents the phase deviation of the FM wave, that is, the maximum departure of the angle (t) from the angle $2\pi f ct$ of the unmodulated carrier.

The FM wave itself is given by

$$s(t) = Ac \cos[2\pi f ct + \text{Q}sin(2\pi f mt)] \ldots\ldots\ldots\ldots\ldots (7)$$

Depending on the value of modulation index β, we may distinguish two cases of frequency modulation. Narrow-band FM for which β is small and Wide-band FM for which β is large, both compared to one radian.

Narrow-Band Frequency modulation

Consider the Single tone FM wave

$$s(t) = Ac \cos[2\pi f ct + \text{Q}sin(2\pi f mt)] \ldots\ldots\ldots\ldots (1)$$

Expanding this relation we get

$$s(t) = A_c \cos(2\pi f_c t) \cos[\beta \sin(2\pi f_m t)] - A_c \sin(2\pi f_c t) \sin[\beta \sin(2\pi f_m t)] \ldots\ldots (2)$$

Assuming that the modulation index β is small compared to one radian, we may use the following approximations:

$$\cos[\beta \sin(2\pi f_m t)] \simeq 1$$

and

$$\sin[\beta \sin(2\pi f_m t)] \simeq \beta \sin(2\pi f_m t)$$

Hence, Equation (2) simplifies to

$$s(t) \simeq A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \sin(2\pi f_m t) \ldots\ldots\ldots\ldots (3)$$

Equation (3) defines the approximate form of a narrowband FM signal produced by a sinusoidal modulating signal $A_m \cos(2\pi f_m t)$. From this representation we deduce the modulator shown in block diagram form in Figure . This modulator involves splitting the carrier wave $A_c \cos(2\pi f_c t)$ into two paths. One path is direct; the other path contains a -90 degree phase-shifting network and a product modulator, the combination of which generates a DSB-SC modulated signal. The difference between these two signals produces a narrowband FM signal, but with some distortion.

Ideally, an FM signal has a constant envelope and, for the case of a sinusoidal modulating signal of frequency f_m, the angle $\theta_i(t)$ is also sinusoidal with the same frequency.

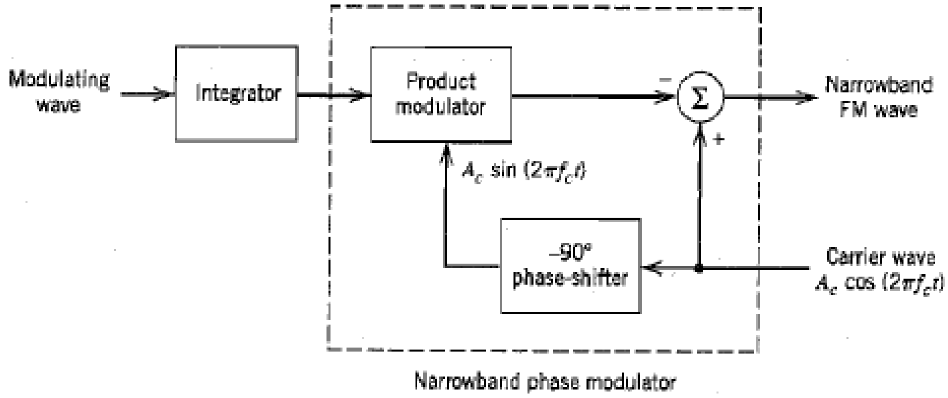


FIGURE Block diagram of a method for generating a narrowband FM signal.

But the modulated signal produced by the narrowband modulator of Figure differs from this ideal condition in two fundamental respects:

1. The envelope contains a *residual* amplitude modulation and, therefore, varies with time.
2. For a sinusoidal modulating wave, the angle $\theta_i(t)$ contains *harmonic distortion* in the form of third- and higher-order harmonics of the modulation frequency f_m.

However, by restricting the modulation index to $\beta \le 0.3$ radians, the effects of residual AM and harmonic PM are limited to negligible levels.

Returning to Equation (3), we may expand it as follows:

$$s(t) = A_c \cos(2\pi f_c t) + \frac{1}{2} \beta A_c \{\cos[2\pi(f_c + f_m)t] - \cos[2\pi(f_c - f_m)t]\} \quad \text{........(4)}$$

This expression is somewhat similar to the corresponding one defining an AM signal, which is as follows:

$$s_{AM}(t) = A_c \cos(2\pi f_c t) + \frac{1}{2} \mu A_c \{\cos[2\pi(f_c + f_m)t] + \cos[2\pi(f_c - f_m)t]\} \quad \text{.........(5)}$$

where μ is the modulation factor of the AM signal. Comparing Equations (**4**) and (**5**), we see that in the case of sinusoidal modulation, the basic difference between an AM signal and a narrowband FM signal is that the algebraic sign of the lower side frequency in the narrowband FM is reversed. Thus, a narrowband FM signal requires essentially the same transmission bandwidth (i.e., $2f_m$) as the AM signal.

We may represent the narrowband FM signal with a phasor diagram as shown in Figure a , where we have used the carrier phasor as reference. We see that the resultant

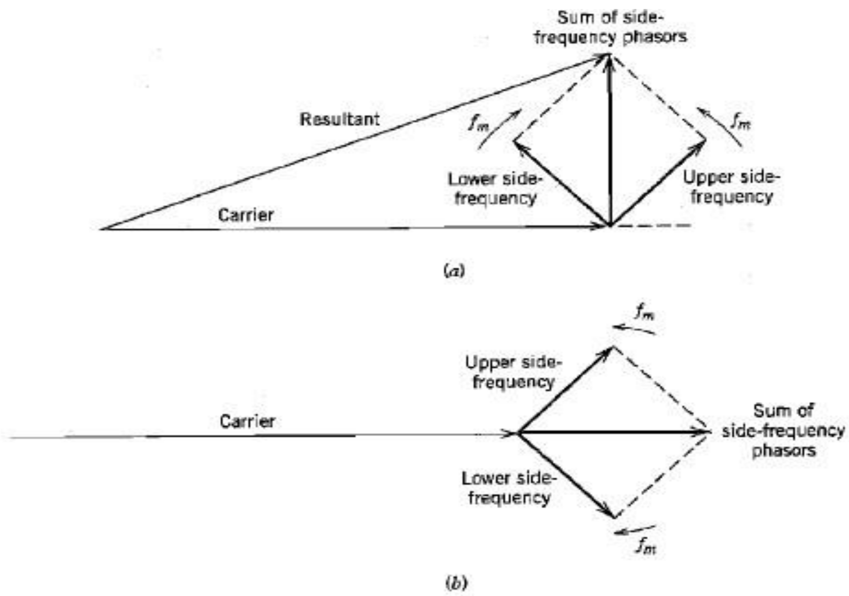


FIGURE A phasor comparison of narrowband FM and AM waves for sinusoidal modulation. (*a*) Narrowband FM wave. (*b*) AM wave.

of the two side-frequency phasors is always at right angles to the carrier phasor. The effect of this is to produce a resultant phasor representing the narrowband FM signal that is approximately of the same amplitude as the carrier phasor, but out of phase with respect to it. This phasor diagram should be contrasted with that of Figure *(b)* , representing an AM signal. In this latter case we see that the resultant phasor representing the AM signal has an amplitude that is different from that of the carrier phasor but always in phase with it.

Wide band frequency Modulation

The spectrum of the signle-tone FM wave of equation

$$s(t) = A_c \cos[2\pi f_c t + Q\sin(2\pi f_m t)] \quad \text{....................(1)}$$

For an arbitrary vale of the modulation index Q is to be determined.

An FM wave produced by a sinusoidal modulating wave as in equation (1) is by itself nonperiodic, unless the carrier frequency fc is an integral multiple of the modualtion frequency fm. Rewriting the equation in the form

$$\begin{aligned} s(t) &= \operatorname{Re}[A_c \exp(j2\pi f_c t + j\beta \sin(2\pi f_m t))] \\ &= \operatorname{Re}[\tilde{s}(t) \exp(j2\pi f_c t)] \quad \text{..........(2)} \end{aligned}$$

where $\tilde{s}(t)$ is the *complex envelope* of the FM signal $s(t)$, defined by

$$\tilde{s}(t) = A_c \exp[j\beta \sin(2\pi f_m t)] \quad \text{...............(3)}$$

$\tilde{s}(t)$ is periodic function of time,with a fundamental frequency equal to the modulation frequency fm. $\tilde{s}(t)$ in the form of complex Fourier series is as follows

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} c_n \exp(j2\pi n f_m t) \quad \text{................(4)}$$

where the complex Fourier coefficient c_n is defined by

$$\begin{aligned} c_n &= f_m \int_{-1/2f_m}^{1/2f_m} \tilde{s}(t) \exp(-j2\pi n f_m t)\, dt \\ &= f_m A_c \int_{-1/2f_m}^{1/2f_m} \exp[j\beta \sin(2\pi f_m t) - j2\pi n f_m t]\, dt \quad \text{............(5)} \end{aligned}$$

Define a new variable:

$$x = 2\pi f_m t \quad \text{.....................(6)}$$

Hence, we may rewrite Equation (5) in the new form

$$c_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} \exp[j(\beta \sin x - nx)]\, dx \quad \text{..............(7)}$$

The integral on the RHS of equation (7) is recognized as the nth order Bessel Function of the first kind and argument Q. This function is commonly denoted by the symbol ***Jn***(Q), that is

$$J_n(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp[j(\beta \sin x - nx)]\, dx \qquad \text{.........(8)}$$

Accordingly, we may reduce Equation (7) to

$$c_n = A_c J_n(\beta) \qquad \text{...............(9)}$$

Substituting Equation (9) in (5), we get, in terms of the Bessel function $J_n(\beta)$, the following expansion for the complex envelope of the FM signal:

$$\tilde{s}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \exp(j2\pi n f_m t) \qquad \text{..............(10)}$$

Next, substituting Equation (10) in (2.), we get

$$s(t) = A_c \cdot \mathrm{Re}\left[\sum_{n=-\infty}^{\infty} J_n(\beta) \exp[j2\pi(f_c + nf_m)t]\right] \qquad \text{.............(11)}$$

Interchanging the order of summation and evaluation of the real part in the right-hand side of Equation (11), we finally get

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + nf_m)t] \qquad \text{................(12)}$$

Equ. (12) is the Fourier series representation of the single-tone FM wave *s(t)* for an arbitrary value of Q.

The discrete spectrum of *s(t)* is obtained by taking the Fourier transform of both sides of equation (12); thus

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)] \text{.............(13)}$$

In the figure below, we have plotted the Bessel function ***Jn***(Q) versus the modulation index Q for different positive integer value of n.

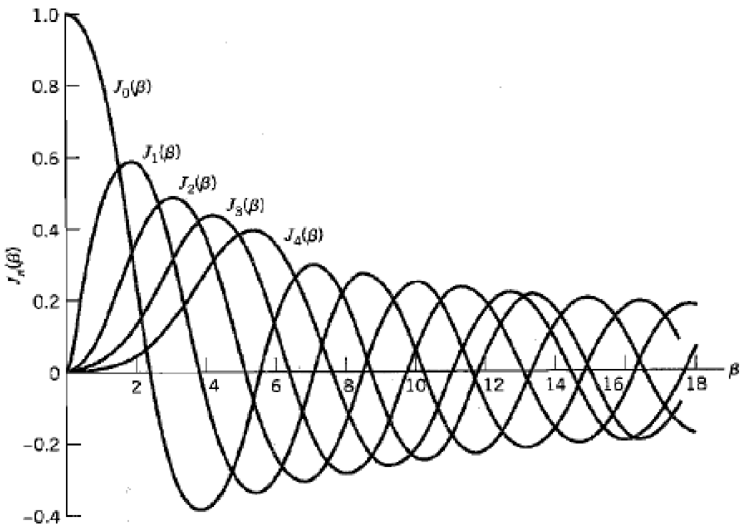


FIGURE Plots of Bessel functions of the first kind for varying order.

Properties of Bessel Function

1. $$J_n(\beta) = (-1)^n J_{-n}(\beta) \quad \text{for all } n \text{, both positive and negative} \qquad \text{.........(14)}$$

2. For small values of the modulation index β, we have

$$\left.\begin{aligned} J_0(\beta) &\simeq 1 \\ J_1(\beta) &\simeq \frac{\beta}{2} \\ J_n(\beta) &\simeq 0, \quad n > 2 \end{aligned}\right\} \qquad \text{...............(15)}$$

3. $$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1 \qquad \text{..................(16)}$$

Thus, using equations (13) through (16) and the curves in the above figure, following observations are made

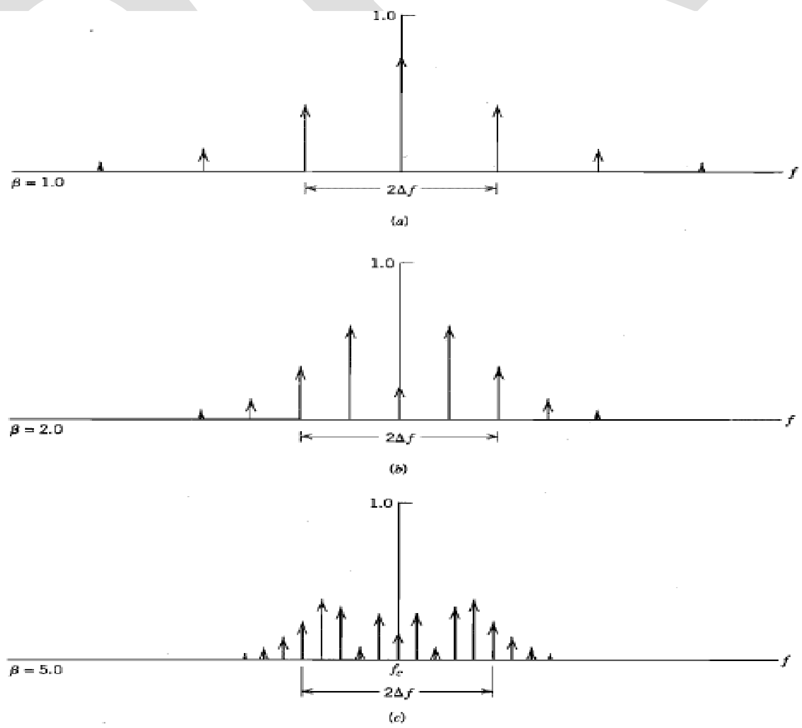
1. The spectrum of an FM signal contains a carrier component and an infinite set of side frequencies located symmetrically on either side of the carrier at frequency separations of f_m, $2f_m$, $3f_m$, $\cdots$. In this respect, the result is unlike that which prevails in an AM system, since in an AM system a sinusoidal modulating signal gives rise to only one pair of side frequencies.
2. For the special case of β small compared with unity, only the Bessel coefficients $J_0(\beta)$ and $J_1(\beta)$ have significant values, so that the FM signal is effectively composed of a carrier and a single pair of side frequencies at $f_c \pm f_m$. This situation corresponds to the special case of narrowband FM that was considered earlier.
3. The amplitude of the carrier component varies with β according to $J_0(\beta)$. That is, unlike an AM signal, the amplitude of the carrier component of an FM signal is dependent on the modulation index β. The physical explanation for this property is that the envelope of an FM signal is constant, so that the average power of such a signal developed across a 1-ohm resistor is also constant, as shown by

$$P = \frac{1}{2} A_c^2 \qquad \text{..................(17)}$$

When the carrier is modulated to generate the FM signal, the power in the side frequencies may appear only at the expense of the power originally in the carrier, thereby making the amplitude of the carrier component dependent on β. Note that the average power of an FM signal may also be determined from Equation (**12**), obtaining

$$P = \frac{1}{2} A_c^2 \sum_{n=-\infty}^{\infty} J_n^2(\beta) \qquad \text{......................(18)}$$

Spectrum Analysis of Sinusoidal FM Wave using Bessel functions



The above figure shows the Discrete amplitude spectra of an FM signal, normalized with respect to the carrier amplitude, for the case of sinusoidal modulation of varying frequency and fixed amplitude. Only the spectra for positive frequencies are shown.

Transmission Bandwidth of FM waves

In theory, an FM signal contains an infinite number of side frequencies so that the bandwidth required to transmit such a signal is similarly infinite in extent. In practice, however, we find that the FM signal is effectively limited to a finite number of significant side frequencies compatible with a specified amount of distortion. We may therefore specify an effective bandwidth required for the transmission of an FM signal. Consider first the

case of an FM signal generated by a single-tone modulating wave of frequency f_m. In such an FM signal, the side frequencies that are separated from the carrier frequency f_c by an amount greater than the frequency deviation Δf decrease rapidly toward zero, so that the bandwidth always exceeds the total frequency excursion, but nevertheless is limited. Specifically, for large values of the modulation index β, the bandwidth approaches, and is only slightly greater than, the total frequency excursion $2\Delta f$ in accordance with the situation shown in Figure 2.25. On the other hand, for small values of the modulation index β, the spectrum of the FM signal is effectively limited to the carrier frequency f_c and one pair of side frequencies at $f_c \pm f_m$, so that the bandwidth approaches $2f_m$. We may thus define an approximate rule for the transmission bandwidth of an FM signal generated by a single-tone modulating signal of frequency f_m as follows:

$$B_T \simeq 2\Delta f + 2f_m = 2\Delta f\left(1 + \frac{1}{\beta}\right)$$

This relation is known as **Carson's rule**